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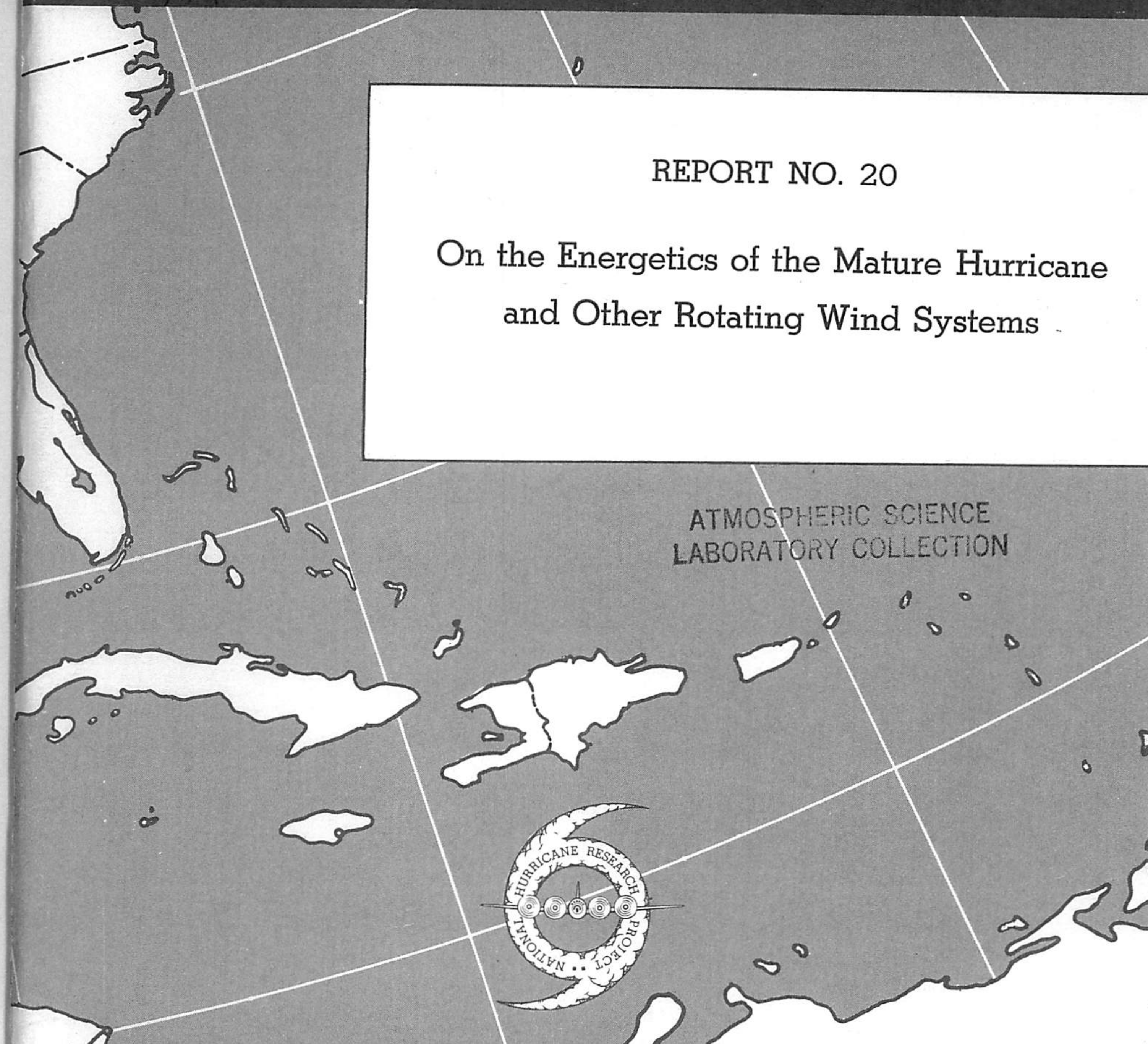

REPORT NO. 20

On the Energetics of the Mature Hurricane and Other Rotating Wind Systems

U. S. DEPARTMENT OF COMMERCE
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by
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ON THE ENERGETICS OF THE MATURE HURRICANE AND OTHER ROTATING WIND SYSTEMS

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ABSTRACT

The first part of this paper is devoted to the formulation and interpretation of the energy equations as they apply to the study of rotating wind systems of interest in meteorology. In a subsequent section the energy cycles of different rotating systems in the atmosphere are discussed. Finally, the roles of "azimuthal-mean motions" and of "horizontal-eddy processes" in the generation of azimuthal-mean kinetic energy in the mature hurricane are examined quantitatively from observational data. As expected, azimuthal-mean motions generate significant amounts of mean rotational kinetic energy within the hurricane. However, interactions with the surrounding atmosphere do not cause organized rotational kinetic energy of the hurricane to be dissipated into that of horizontal eddies, as might be expected. According to the observations, the reverse is the case; namely, that the hurricane circulation feeds on the energy of the horizontal eddies.

1. INTRODUCTION

The large-scale motions of the atmosphere are characterized largely by the fact that they are organized into systems which possess rotation about vertical axes. While it is generally accepted that this rotation is maintained in the end as a result of the differential heating of the atmosphere and the rotation of the earth, a fundamental problem of meteorology has been that of ascertaining the physical processes which operate to convert thermodynamical energy into the kinetic energy of organized rotation. In recent years this problem has received considerable attention, and substantial progress has been made toward a better understanding of the dynamics of rotating fluid systems in general. The theoretical work of Kuo [10,11,12,13,14], Davies [2,3], and Lorenz [16], and the experiments of Fultz [4,5] and Hide [7], dealing with the behavior of water in a cylindrical vessel under the influence of differential heating and rotation, have contributed significantly to this understanding.

The purpose of the present article is threefold: (1) to formulate and interpret a system of equations suitable for studying the energetics of rotating wind systems of meteorological interest; (2) to explore, with the aid of these equations, the various ways in which rotational motion can be generated and maintained in the atmosphere; and (3) to examine quantitatively from observational data the particular way in which rotational motion is maintained in the case of the mature hurricane. The basic principle around which the present discussion is organized is the law of conservation of energy. Previous observational studies of the dynamics of the mature hurricane conducted by the writer [18,19] have dealt with the balance of angular momentum about the axis of rotation of the hurricane. Whereas the principle of conservation of momentum provides a simple framework in which to study the mechanical aspects of the problem, the principle of conservation of energy provides a convenient link between the mechanics and the thermodynamics of the problem.

A fundamental question which arises in connection with the energetics of rotating wind systems concerns the extent to which such systems derive their kinetic energy directly from potential and internal energy, and the extent to which they give energy to, or receive energy from, other systems in their environment. The work of Reynolds [21], dealing with the stability of turbulent flow in an incompressible fluid, provides a basis for attacking questions of this nature in a straightforward manner. By resolving the fluid velocity into a mean and a deviation therefrom, Reynolds derived energy equations which govern the interactions between a "basic current" and finite eddy disturbances in the flow. More recently, Lorenz [17] has considered a similar partitioning of available potential energy by resolving the temperature field into a mean and a deviation therefrom.

The present development follows the line of attack set forth by Reynolds, with the exception that no assumptions are made here regarding the compressibility of the fluid. As a result, it will be possible to study conversions between kinetic and potential plus internal energy, as well as conversions between the kinetic energy of the basic current and the kinetic energy of the eddies. The energy equations will be derived relative to a system of spherical polar coordinates $(r, \Delta\phi, \theta)$ with origin located at the center of the earth, where r is linear distance from the origin measured along a vertical axis which coincides with the axis of a rotating wind system, $\Delta\phi$ is angular distance from this axis, and θ is azimuth measured positive in the counterclockwise sense. The procedure to be followed consists in resolving each of the independent variables in the mechanical energy equation into a mean with respect to θ and a horizontal-eddy component defined as the deviation from this mean. With the use of this resolution we shall then derive equations for the time rate of change of azimuthal-mean and horizontal-eddy forms of kinetic energy. The former may be thought of as properties of the rotating system under consideration, and the latter as properties of neighboring systems. Before proceeding with these developments, however, we shall begin with a discussion and interpretation of the equations for total energy.

2. EQUATIONS FOR TOTAL ENERGY

The following notation will be used throughout the remainder of the paper:

Symbol	Definition
(x, y, z)	linear distances in the θ, $-\Delta\phi$, and r directions, respectively. $r = a + z$, where a is the radius of the earth.

(dx, dy, dz)	$((r \sin \Delta\phi)d\theta, \; -rd(\Delta\phi), \; dr)$
(C_T, C_n, w)	the components of the velocity in the x, y, and z directions, respectively.
(D_x, D_y, D_z)	the components of the viscous force per unit mass in the x, y, and z directions, respectively.
d	$\rho C_T D_x + \rho C_n D_y$
ϵ	$r^2 \sin \Delta\phi$
t	time
R	gas constant for dry air
C_V	heat capacity of dry air at constant volume
Q	rate of external heating per unit mass
$\lvert\Psi\rvert$	rate of generation of internal energy per unit volume by viscosity
g	acceleration of gravity
p, T, ρ	pressure, temperature, and density, respectively
K, Φ, I	horizontal kinetic $\left(\frac{C_T^2 + C_n^2}{2}\right)$, potential (gz), and internal $(C_V T)$ energy per unit mass, respectively
ϕ_0	latitude of the vertical axis along which r is measured
ω	angular velocity of the earth
ω_{ϕ_0}	$\omega \sin \phi_0$
f_n	$2\,\omega_{\phi_0} \cos \Delta\phi$

In terms of the above notation we may write the hydrodynamical equations in the form,

$$\rho \frac{dC_T}{dt} = \frac{C_T(\rho C_n) \cot \Delta\phi}{r} - \frac{C_T(\rho w)}{r} + (2\omega_{\phi_0} \cos \Delta\phi)\, \rho C_n - (2\omega_{\phi_0} \sin \Delta\phi)\, \rho w - \frac{\partial p}{\partial x} + \rho D_x \tag{1}$$

$$\rho \frac{dC_n}{dt} = - \frac{C_T (\rho C_T) \cot \Delta\phi}{r} - \frac{C_n (\rho w)}{r} - (2\omega_{\phi_0} \cos \Delta\phi) \rho C_T - \frac{\partial p}{\partial y} + \rho D_y \quad , \tag{2}$$

$$\rho \frac{dw}{dt} = \frac{C_T(\rho C_T)}{r} + \frac{C_n(\rho C_n)}{r} + (2\omega_{\phi_0} \sin \Delta\phi) \rho C_T - \rho g - \frac{\partial p}{\partial z} + \rho D_z \quad , \tag{3}$$

$$\frac{\partial \rho}{\partial t} = - \frac{1}{\epsilon} \left(\frac{\partial \epsilon \rho C_T}{\partial x} + \frac{\partial \epsilon \rho C_n}{\partial y} + \frac{\partial \epsilon \rho w}{\partial z} \right) \quad , \tag{4}$$

$$\rho \frac{dC_V T}{dt} = - \frac{p}{\epsilon} \left(\frac{\partial \epsilon C_T}{\partial x} + \frac{\partial \epsilon C_n}{\partial y} + \frac{\partial \epsilon w}{\partial z} \right) + \rho Q + |\Psi| \quad , \tag{5}$$

$$p = \rho R T \, . \tag{6}$$

The first four terms on the right hand side of (1) measure the rate at which the Θ - component of the relative linear momentum of a fluid element increases when the element moves closer to the axis of rotation under conservation of angular momentum. The first three terms on the right hand side of equations (2) and (3) have a similar interpretation. These terms may be thought of as fictitious forces.

Assuming hydrostatic equilibrium, and neglecting the terms $\frac{C_T(\rho w)}{r}$, $\frac{C_n(\rho w)}{r}$, and $(2\omega_{\phi_0} \sin \Delta\phi)(\rho w)$ in comparison with $\frac{C_T(\rho C_n) \cot \Delta\phi}{r}$, $\frac{C_T(\rho C_T) \cot \Delta\phi}{r}$, and $(2\omega_{\phi_0} \cos \Delta\phi)(\rho C_n)$, respectively, we may rewrite equations (1), (2), and (3) as follows:

$$\rho \frac{dC_T}{dt} = \frac{C_T(\rho C_n) \cot \Delta\phi}{r} + (2\omega_{\phi_0} \cos \Delta\phi)(\rho C_n) - \frac{\partial p}{\partial x} + \rho D_x \tag{7}$$

$$\rho \frac{dC_n}{dt} = - \frac{C_T(\rho C_T) \cot \Delta\phi}{r} - (2\omega_{\phi_0} \cos \Delta\phi)(\rho C_T) - \frac{\partial p}{\partial y} + \rho D_y \tag{8}$$

$$\rho g = - \frac{\partial p}{\partial z} \tag{9}$$

Multiplying (7) by C_T, (8) by C_n, and (9) by w, we get

$$\rho \frac{d}{dt}\left(\frac{C_T^2}{2}\right) = \frac{C_T C_T(\rho C_n) \cot \Delta\phi}{r} + (2\omega_{\phi_0} \cos \Delta\phi)\, C_T(\rho C_n) - C_T \frac{\partial p}{\partial x} + \rho C_T D_x \tag{10}$$

$$\rho \frac{d}{dt}\left(\frac{C_n^2}{2}\right) = -\frac{C_T C_T(\rho C_n) \cot \Delta\phi}{r} - (2\omega_{\phi_0} \cos \Delta\phi)\, C_T(\rho C_n) - C_n \frac{\partial p}{\partial y} + \rho C_n D_y \tag{11}$$

$$\rho \frac{d\Phi}{dt} = - w\frac{\partial p}{\partial z} \quad . \tag{12}$$

It will be noted that the terms, $\frac{C_T C_T(\rho C_n) \cot \Delta\phi}{r}$, and $(2\omega_{\phi_0} \cos \Delta\phi)\, C_T' (\rho C_n)$ appear in equations (10) and (11) with opposite signs. These terms measure the rate of positive generation of kinetic energy of tangential motion, and the rate of negative generation of kinetic energy of normal motion, by the fictitious forces. Their sum measures the rate of conversion between the two components of horizontal kinetic energy, and cannot, therefore, effect a change in the total kinetic energy of a fluid element. (It should be remarked that a similar coupling with the potential energy equation is eliminated by the assumption of hydrostatic equilibrium and the neglect, in the equations of motion, of the fictitious forces involving the vertical component of velocity.) By combining (10) and (11) we eliminate the fictitious work terms and arrive at the following expression for the rate of change of the horizontal kinetic energy on a fluid element,

$$\rho \frac{dK}{dt} = -\left(C_T \frac{\partial p}{\partial x} + C_n \frac{\partial p}{\partial y}\right) + d \tag{13}$$

Integrating equations (13), (5), and (12) over the volume, V, which is contained between the two conical surfaces $\Delta\phi = (\Delta\phi)_a$ and $\Delta\phi = (\Delta\phi)_b$ and between the two levels $z = z_1$ and $z = z_2$ (which lie at approximate pressures $p = p_1$ and $p = p_2$), and making use of (4) we obtain,

$$\frac{\partial K}{\partial t} = -\int_V \left(C_T \frac{\partial p}{\partial x} + C_n \frac{\partial p}{\partial y}\right) dV + d \quad , \tag{14}$$

$$\frac{\partial \mathbf{I}}{\partial t} = - \int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon c_T}{\partial x} + \frac{\partial \epsilon c_n}{\partial y} \right) dV + |\mathbf{\Psi}| - \int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon w}{\partial z} \right) dV + \mathbf{H} \,, \tag{15}$$

$$\frac{\partial \mathbf{\Phi}}{\partial t} = - \int_V w \frac{\partial p}{\partial z} \, dV \quad , \tag{16}$$

in which the various terms have the following interpretations[1]:

$\mathbf{K} = \int_V \rho K dV$ is the total kinetic energy contained within the volume.

$\mathbf{I} = \int_V \rho I dV$ is the total internal energy contained within the volume.

$\mathbf{\Phi} = \int_V \rho \bar{\phi} dV$ is the total potential energy contained within the volume.

$\mathbf{d} = \int_V d dV$ is the total rate of generation of kinetic energy within the volume by friction.

$|\mathbf{\Psi}| = \int_V |\Psi| dV$ is the total rate of generation of internal energy within the volume by friction.

$\mathbf{H} = \int_V \rho Q dV$ is the total flux of heat into the volume.

$- \int_V \left(c_T \frac{\partial p}{\partial x} + c_n \frac{\partial p}{\partial x} \right) dV$ is the total rate of generation of kinetic energy within the volume by the horizontal pressure forces.

$- \int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon c_T}{\partial x} + \frac{\partial \epsilon c_n}{\partial y} \right) dV$ is the total rate of generation of internal energy within the volume by horizontal compressions against pressure forces.

$- \int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon w}{\partial z} \right) dV$ is the total rate of generation of internal energy within the volume by vertical compressions against pressure forces.

$- \int_V w \frac{\partial p}{\partial z} dV$ is the total rate of generation of potential energy within the volume by the vertical pressure forces.

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1 The meanings of the terms "generation" and "conversion" were discussed elsewhere (Pfeffer [20]).

To the above equations we should add the identities,

$$\int_{S_a} pC_n dS_a + \int_{S_b} pC_n dS_b = - \int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon C_T}{\partial x} + \frac{\partial \epsilon C_n}{\partial y} \right) dV -$$

$$\int_V \left(C_T \frac{\partial p}{\partial x} + C_T \frac{\partial p}{\partial y} \right) dV \qquad (17)$$

$$- \int_{S_1} pwdS_1 + \int_{S_2} pwdS_2 = - \int_V \frac{p}{\epsilon} \frac{\partial \epsilon w}{\partial z} dV - \int_V w \frac{\partial p}{\partial z} dV \quad , \qquad (18)$$

$$\mathbf{S} = |\mathbf{\Psi}| + \mathbf{d} \quad , \qquad (19)$$

where S_a and S_b represent area measured along the conical walls $\Delta\phi = (\Delta\phi)_a$ and $\Delta\phi = (\Delta\phi)_b$, respectively, and S_1 and S_2 represent area measured along the horizontal boundaries at z_1 and z_2, respectively. If C_n vanishes everywhere on the boundary, then

$$\int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon C_T}{\partial z} + \frac{\partial \epsilon C_n}{\partial y} \right) dV = - \int_V \left(C_T \frac{\partial p}{\partial x} + C_T \frac{\partial p}{\partial y} \right) dV \; ,$$

and if $\mathbf{d}$ and $|\mathbf{\Psi}|$ are each zero, either of these integrals may be taken as the rate of conversion between kinetic and internal energy. Similarly if w vanishes everywhere on the boundary then

$$\int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon w}{\partial z} \right) dV = - \int_V w \frac{\partial p}{\partial z} dV \quad ,$$

and either of these integrals may be taken as the rate of conversion between internal and potential energy.

It is interesting to note that to the extent that the atmosphere is in hydrostatic equilibrium the following relationship (see Haurwitz [6]) holds,

$$\mathbf{\Phi} = (\lambda + 1) \quad , \qquad (20)$$

where $\lambda = \frac{C_p}{C_v}$, and the volume integrals in the definitions of $\mathbf{\Phi}$ and $\mathbf{I}$ are assumed to extend through the entire depth of the atmosphere. Using (20) we can show that for a closed system in which the non-conservative terms $\mathbf{d}$, $|\mathbf{\Psi}|$, and $\mathbf{H}$ are each zero, the rate of conversion between internal and kinetic energy must be related to the rate of conversion between potential and internal energy in the following manner:

$$\int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon c_T}{\partial x} + \frac{\partial \epsilon c_n}{\partial y} \right) dV = - \left(\frac{\lambda}{\lambda - 1} \right) \int_V \frac{p}{\epsilon} \frac{\partial \epsilon w}{\partial z} dV \quad . \qquad (21)$$

Thus, if internal energy is being converted to kinetic energy, a certain fraction of the internal energy loss is simultaneously replenished by a conversion from potential energy. If on the other hand, kinetic energy is being converted to internal energy a certain fraction of the internal energy gain spills over into potential energy. Because of this relationship it is customary to regard the sum of internal and potential energy as one form of energy. The energy equations then reduce to,

$$\frac{\partial \mathbf{K}}{\partial t} = - \int_V \left(c_T \frac{\partial p}{\partial x} + c_n \frac{\partial p}{\partial y} \right) dV + \mathbf{d} \qquad (22)$$

$$\frac{\partial \mathbf{P}}{\partial t} = - \int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon c_T}{\partial x} + \frac{\partial \epsilon c_n}{\partial y} \right) dV + \mathbf{\Psi} + \mathbf{H} \qquad (23)$$

where $\mathbf{P} = \mathbf{\Phi} + \mathbf{I}$. The term

$$- \int_V \frac{p}{\epsilon} \left(\frac{\partial \epsilon c_T}{\partial x} + \frac{\partial \epsilon c_n}{\partial y} \right) dV \equiv - \int_V p \nabla_3 \cdot \mathbb{V}_3 dV - \int_V w \frac{\partial p}{\partial z} dV$$

is now regarded as the rate of generation of internal plus potential energy by the pressure forces. This interpretation is justified on the grounds that the integrals, $- \int_V p \nabla_3 \cdot \mathbb{V}_3 dV$ and $- \int_V w \frac{\partial p}{\partial z} dV$, individually measure the rate of generation of internal energy by compression against the pressure forces, and the rate of generation of potential energy by the vertical pressure force.

3. EQUATIONS FOR THE KINETIC ENERGY OF AZIMUTHAL-MEAN FLOW

Equations (7) and (8) may be written with the aid of (4) in the following forms,

$$\frac{\partial \rho c_T}{\partial t} = - \frac{1}{\epsilon} \left\{ \frac{\partial \epsilon c_T \rho c_T}{\partial x} + \frac{\partial \epsilon c_T \rho c_n}{\partial y} + \frac{\partial \epsilon c_T \rho w}{\partial z} \right\} - \frac{\partial p}{\partial x} + \rho D_x +$$

$$\frac{c_T (\rho c_n) \cot \Delta \phi}{r} + (2\omega_{\phi_0} \cos \Delta \phi) \rho c_n \quad , \qquad (24)$$

$$\frac{\partial \rho C_n}{\partial t} = -\frac{1}{\epsilon}\left(\frac{\partial \epsilon C_n \rho C_T}{\partial x} + \frac{\partial \epsilon C_n \rho C_n}{\partial y} + \frac{\partial \epsilon C_n \rho w}{} \right) - \frac{\partial p}{\partial y} + \rho D_y -$$

$$\frac{C_T(\rho C_T) \cot \Delta\phi}{r} + (2\omega_{\phi_0} \cos \Delta\phi)\, \rho C_T \quad . \qquad (25)$$

Applying the average,

$$[\,(\quad)\,] \equiv \frac{1}{2\pi} \oint (\quad)\, d\theta \quad , \qquad (26)$$

to (24) and (25) and making use of the identity,

$$[C_1 C_2] = [C_1]\,[C_2] + [C_1{}'\, C_2{}'] \qquad (27)$$

where the subscripts are dummy indices we have ,

$$\frac{\partial \rho [C_T]}{\partial t} = -\frac{1}{\epsilon}\left(\frac{\partial \rho\, \epsilon\, [C_T][C_n]}{\partial y} + \frac{\partial \rho\, \epsilon\, [C_T][w]}{\partial z} + \frac{\partial \rho\, \epsilon\, [C_T{}'C_n{}']}{\partial y} + \frac{\partial \rho\, \epsilon\, [C_T{}'w']}{\partial z} \right) +$$

$$\frac{\rho [C_T][C_n] \cot \Delta\phi}{r} + \frac{\rho [C_T{}'C_n{}'] \cot \Delta\phi}{r} + (2\omega_{\phi_0} \cos \Delta\phi) \rho [C_n] + \rho [D_x] \quad (28)$$

$$\frac{\partial \rho [C_n]}{\partial t} = -\frac{1}{\epsilon}\left(\frac{\partial \rho\, \epsilon\, [C_n][C_n]}{\partial y} + \frac{\partial \rho\, \epsilon\, [C_n][w]}{\partial z} + \frac{\partial \rho\, \epsilon [C_n{}'C_n{}']}{\partial y} + \frac{\partial \rho\, \epsilon [C_n{}'w']}{\partial z} \right) -$$

$$\frac{\rho [C_T][C_T] \cot \Delta\phi}{r} - \frac{\rho [C_T{}'C_T{}'] \cot \Delta\phi}{r} - (2\omega_{\phi_0} \cos \Delta\phi)\, \rho [C_T] -$$

$$\frac{\partial [p]}{\partial y} + \rho [D_y] \qquad (29)$$

where $(\quad)' \equiv (\quad) - [(\quad)]$, and we have assumed that $\rho \neq \rho(\theta)$. Multiplying (28) by $[C_T]$ and (29) by $[C_n]$ and making use of (4) once again we get,

$$\frac{\partial}{\partial t}\left(\frac{\rho[C_T]^2}{2}\right) = -\frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}\left(\epsilon\,\frac{\rho[C_T]^2}{2}\,[C_n]\right) + \frac{\partial}{\partial z}\left(\epsilon\,\frac{\rho[C_T]^2}{2}\,[w]\right)\right\} -$$

$$\frac{[C_T]}{\epsilon}\left\{\frac{\partial}{\partial y}(\rho\,\epsilon\,[C_T'C_n']) + \frac{\partial}{\partial z}(\rho\,\epsilon\,[C_T'w'])\right\} + \rho[C_T][D_x] +$$

$$\frac{\rho[C_T][C_T][C_n]\cot\Delta\phi}{r} + \frac{\rho[C_T][C_T'C_n']\cot\Delta\phi}{r} + \rho f_n[C_T][C_n]\,, \qquad (30)$$

$$\frac{\partial}{\partial t}\left(\frac{\rho[C_n]^2}{2}\right) = \frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}\left(\epsilon\,\frac{\rho[C_n]^2}{2}[C_n]\right) + \frac{\partial}{\partial z}\left(\epsilon\,\frac{\rho[C_n]^2}{2}\,[w]\right)\right\} - [C_n]\frac{\partial[p]}{\partial y} -$$

$$\frac{[C_n]}{\epsilon}\left\{\frac{\partial}{\partial y}(\rho\,\epsilon\,[C_n'C_n']) + \frac{\partial}{\partial z}(\rho\,\epsilon\,[C_n'w'])\right\} + \rho[C_n][D_y] -$$

$$\frac{\rho[C_T][C_T][C_n]\cot\Delta\phi}{r} - \frac{\rho[C_n][C_T'C_T']\cot\Delta\phi}{r} - \rho f_n[C_T][C_n]\;. \qquad (31)$$

Integrating (30) and (31) over V we obtain finally,

$$\frac{\partial[\mathbf{K}]}{\partial t} = \int_{S_a}\rho\,\frac{[C_T]^2}{2}\,C_n dS_a - \int_{S_b}\rho\,\frac{[C_T]^2}{2}\,C_n dS_b + \int_{S_1}\rho\,\frac{[C_T]^2}{2}\,w dS_1 -$$

$$\int_{S_2}\rho\,\frac{[C_T]^2}{2}\,w dS_2 - \int_V\frac{[C_T]}{\epsilon}\left\{\frac{\partial}{\partial y}(\rho\,\epsilon\,[C_T'C_n']) + \frac{\partial}{\partial z}(\rho\,\epsilon\,[C_T'w'])\right\}dV +$$

$$\int_V\rho[C_T][D_x]dV + \int_V\frac{\rho[C_T][C_T][C_n]\cot\Delta\phi}{r}\,dV +$$

$$\int_V\frac{\rho[C_T][C_T'C_n']\cot\Delta\phi}{r}\,dV + \int_V\rho f_n[C_T][C_n]\,dV\;, \qquad (32)$$

$$\frac{\partial[\mathbf{k}]}{\partial t} = \int_{S_a} \rho \frac{[C_n]^2}{2} C_n dS_a - \int_{S_b} \rho \frac{[C_n]^2}{2} C_n dS_b + \int_{S_1} \rho \frac{[C_n]^2}{2} w dS_1 -$$

$$\int_{S_2} \rho \frac{[C_n]^2}{2} w dS_2 - \int_V [C_n] \frac{\partial[p]}{\partial y} dV - \int_V \frac{[C_n]}{\epsilon} \left\{ \frac{\partial}{\partial y} (\rho \epsilon [C_n' C_n']) + \right.$$

$$\left. \frac{\partial}{\partial z} (\rho \epsilon [C_n' w']) \right\} dV + \int_V \rho [C_n][D_y] dV - \int_V \frac{\rho [C_T][C_T][C_n] \cot \Delta\phi}{r} dV -$$

$$\int_V \frac{\rho [C_n][C_T' C_T'] \cot \Delta\phi}{r} dV - \int_V \rho f_n [C_T][C_n] dV \quad , \tag{33}$$

in which the various terms have the following interpretations:

$[\mathbf{K}] \equiv \int_V \rho \frac{[C_T]^2}{2} dV$ is the total kinetic energy of the azimuthal-mean tangential flow contained within the volume.

$[\mathbf{k}] \equiv \int_V \rho \frac{[C_n]^2}{2} dV$ is the total kinetic energy of the azimuthal-mean normal flow contained within the volume.

$$\int_{S_a} \frac{\rho [C_T]^2}{2} C_n dS_a - \int_{S_b} \frac{\rho [C_T]^2}{2} C_n dS_b + \int_{S_1} \frac{\rho [C_T]^2}{2} w dS_1 - \int_{S_2} \frac{\rho [C_T]^2}{2} w dS_2$$

is the net transport into the volume of the kinetic energy of azimuthal-mean tangential flow.

$$\int_{S_a} \frac{\rho [C_n]^2}{2} C_n dS_a - \int_{S_b} \frac{\rho [C_n]^2}{2} C_n dS_b + \int_{S_1} \frac{\rho [C_n]^2}{2} w dS_1 - \int_{S_2} \frac{\rho [C_n]^2}{2} w dS_2$$

is the net transport into the volume of the kinetic energy of azimuthal-mean normal flow.

$- \int_V \frac{[C_T]}{\epsilon} \left\{ \frac{\partial}{\partial y} (\rho \epsilon [C_T' C_n']) + \frac{\partial}{\partial z} (\rho \epsilon [C_T' w']) \right\} dV$ is the total rate of generation of azimuthal-mean tangential kinetic energy within the volume by the horizontal-eddy stresses. The integrand is given by the product of the azimuthal-mean tangential motion of an elementary annular ring with the rate at which eddy processes accumulate linear tangential momentum within the ring. This term is somewhat analogous to the pressure work term, $-\int_V [C_n]\frac{\partial p}{\partial y}\, dV$, in that the integrand represents a net stress on the ring in the direction of the mean motion of the ring. In the case of the pressure work term, the (normal) pressure stress is involved, whereas here the horizontal-eddy stress in the tangential direction is involved.

$$-\int_V \frac{[C_n]}{\epsilon}\left\{\frac{\partial}{\partial y}(\rho\,\epsilon\,[C_n'C_n']) + \frac{\partial}{\partial z}(\rho\,\epsilon\,[C_n'w'])\right\} dV$$ is the total rate of generation of azimuthal-mean normal kinetic energy within the volume by the horizontal-eddy stresses. The integrand is given by the product of the azimuthal-mean normal motion of an elementary annular ring with the rate at which horizontal-eddy processes accumulate linear normal momentum within the ring. This term is closely analogous to the pressure work term, $-\int_V [C_n]\frac{\partial p}{\partial y} dV$, in that the integrand of each represents a net normal stress on the ring in the direction of the azimuthal-mean normal motion of the ring.

$$\int_V \rho[C_T][C_n]\left(\frac{[C_T]\cot\Delta\phi}{r} + f_n\right) dV$$ is the total rate of generation of azimuthal-mean tangential kinetic energy within the volume, by the fictitious forces, through the agency of azimuthal-mean motions.

$$-\int_V \rho[C_T][C_T]\left(\frac{[C_n]\cot\Delta\phi}{r} + f_n\right) dV$$ is the total rate of generation of azimuthal-mean normal kinetic energy within the volume by the fictitious forces, through the agency of azimuthal-mean motions.

$$\int_V \rho[C_T'C_n']\left(\frac{[C_T]\cot\Delta\phi}{r}\right) dV$$ is the total rate of generation of azimuthal-mean tangential kinetic energy within the volume by the fictitious forces, through the agency of horizontal-eddy motions.

$$-\int_V \rho[C_T'C_T']\left(\frac{[C_n]\cot\Delta\phi}{r}\right) dV$$ is the total rate of generation of azimuthal-mean normal kinetic energy within the volume by the fictitious forces, through the agency of horizontal-eddy motions.

$\int_V \rho[C_T][D_x]dV$ is the total rate of generation of azimuthal-mean tangential kinetic energy within the volume by friction.

$\int_V \rho[C_n][D_x]dV$ is the total rate of generation of azimuthal-mean normal kinetic energy within the volume by friction.

$-\int_V [C_n] \frac{\partial[p]}{\partial y} dV$ is the total rate of generation of azimuthal-mean normal kinetic energy within the volume by the horizontal pressure forces. It will be noted that there is no corresponding term in the equation for $[\mathbf{K}]$.

4. EQUATIONS FOR THE KINETIC ENERGY OF HORIZONTAL-EDDY FLOW

Equations (10) and (11) may be written with the aid of (4) in the following forms,

$$\frac{\partial}{\partial t}\left(\frac{\rho C_T^2}{2}\right) = -\frac{1}{\epsilon}\left\{\frac{\partial}{\partial x}\left(\frac{\rho C_T^2}{2}\epsilon C_T\right) + \frac{\partial}{\partial y}\left(\frac{\rho C_T^2}{2}\epsilon C_n\right) + \frac{\partial}{\partial z}\left(\frac{\rho C_T^2}{2}\epsilon w\right)\right\} +$$
$$\rho f_n C_T C_n + \frac{\rho C_T^2 C_n \cot \Delta\phi}{r} + C_T \frac{\partial p}{\partial x} + \rho C_T D_x \quad , \qquad (34)$$

$$\frac{\partial}{\partial t}\left(\frac{\rho C_n^2}{2}\right) = -\frac{1}{\epsilon}\left\{\frac{\partial}{\partial x}\left(\frac{\rho C_n^2}{2}\epsilon C_T\right) + \frac{\partial}{\partial y}\left(\frac{\rho C_n^2}{2}\epsilon C_n\right) +\right.$$
$$\left.\frac{\partial}{\partial z}\left(\frac{\rho C_n^2}{2}\epsilon w\right)\right\} - \rho f_n C_T C_n - \frac{\rho C_T^2 C_n \cot \Delta\phi}{r} + C_T \frac{\partial p}{\partial y} + \rho C_n D_y \,. \qquad (35)$$

Applying (26) to (34) and (35) and making use of the identities,

$$[C_1^2 C_2] = [C_1][C_1][C_2] + 2[C_1][C_1'C_2'] + [C_1'C_1'C_2] \qquad (36)$$

and

$$[C_1^2 C_2] = [C_1][C_1][C_2] + [C_1][C_1'C_2'] + [C_1'C_1'][C_2] +$$
$$[C_1 C_1' C_2'] \qquad (37)$$

where the subscripts are dummy indices, we have

$$\frac{\partial}{\partial t}\left(\frac{\rho[C_T{}^2]}{2}\right) = -\frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}\left(\rho\,\epsilon\,\frac{[C_T]^2}{2}[C_n]\right) + \frac{\partial}{\partial z}\left(\rho\,\epsilon\,\frac{[C_T]^2}{2}[C_n]\right)\right\} -$$

$$\frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}(\rho\,\epsilon\,[C_T][C_T{}'C_n{}']) + \frac{\partial}{\partial z}(\rho\,\epsilon\,[C_T][C_T{}'w'])\right\} -$$

$$\frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}\left(\rho\,\epsilon\left[\frac{(C_T{}')^2}{2}C_n\right]\right) + \frac{\partial}{\partial z}\left(\rho\,\epsilon\left[\frac{(C_T{}')^2}{2}w\right]\right)\right\} +$$

$$\rho[C_T][C_n]\left(\frac{[C_T]\cot\Delta\phi}{r} + f_n\right) + \rho[C_T{}'C_n{}']\left(\frac{[C_T]\cot\Delta\phi}{r}\right) +$$

$$\rho f_n[C_T{}'C_n{}'] + \rho[C_T{}'C_T]\left(\frac{[C_n]\cot\Delta\phi}{r}\right. + \rho[C_TC_T{}'C_n{}']\,\frac{\cot\Delta\phi}{r} -$$

$$[C_T{}'\,\frac{\partial p'}{\partial x}] + \rho[C_T][D_x] + \rho[C_TD_x{}'] \tag{38}$$

and

$$\frac{\partial}{\partial t}\left(\frac{\rho[C_n{}^2]}{2}\right) = -\frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}\left(\epsilon\,\frac{\rho[C_n]^2}{2}[C_n]\right) + \frac{\partial}{\partial z}\left(\epsilon\,\frac{\rho[C_n]^2}{2}[w]\right)\right\} -$$

$$\frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}(\rho\,\epsilon\,[C_n][C_n{}'C_n]) + \frac{\partial}{\partial z}(\rho\,\epsilon\,[C_n][C_n{}'w'])\right\} -$$

$$\frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}\left(\epsilon\left[\frac{\rho(C_n{}')^2}{2}C_n\right]\right) + \frac{\partial}{\partial z}\left(\epsilon\left[\frac{\rho(C_n{}')^2}{2}w\right]\right)\right\} -$$

$$\rho[C_T][C_n]\left(\frac{[C_T]\cot\Delta\phi}{r} + f_n\right) - \rho[C_T{}'C_n{}']\left(\frac{[C_T]\cot\Delta\phi}{r}\right) -$$

$$\rho f_n[C_T{}'C_n{}'] - \rho[C_T{}'C_T{}']\left(\frac{[C_n]\cot\Delta\phi}{r}\right) - \rho[C_TC_T{}'C_n{}']\,\frac{\cot\Delta\phi}{r} -$$

$$[C_n]\,\frac{\partial[p]}{\partial y} - [C_n{}'\,\frac{\partial p'}{\partial y}] + \rho[C_n][D_y] + \rho[C_n{}'D_y{}'] \tag{39}$$

where we have assumed again that $\rho \neq \rho(\Theta)$. Subtracting (30) and (31) from (38 and (39) respectively, and making use of the identities,

$$\frac{\partial}{\partial\eta}(\rho\,\epsilon\,[C_1][C_1'C_2']) = [C_1]\,\frac{\partial\rho\,\epsilon\,[C_1'C_2']}{\partial\eta} + \rho\,\epsilon\,[C_1'C_2']\,\frac{\partial[C_1]}{\partial\eta} \tag{40}$$

and

$$[C_1{}^2] = [C_1]^2 + [(C_1')^2] \quad , \tag{41}$$

where the subscripts are dummy indices, and η is a dummy variable, we obtain,

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\rho[(C_T')^2]}{2}\right) = &-\frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}\left(\rho\,\epsilon\left[\frac{(C_T')^2}{2}C_n\right]\right) + \frac{\partial}{\partial z}\left(\rho\,\epsilon\left[\frac{(C_T')^2}{2}w\right]\right)\right\} - \\ &\rho[C_T'C_n']\frac{\partial[C_T]}{\partial y} - \rho[C_T'w']\frac{\partial[C_T]}{\partial z} + \rho f_n[C_T'C_n'] + \\ &\rho[C_T'C_T']\frac{[C_n]\cot\Delta\phi}{r} + \rho[C_T'C_n'C_T]\frac{\cot\Delta\phi}{r} - \\ &[C_T'\frac{\partial p'}{\partial x}] + \rho[C_T'D_x']\end{aligned} \tag{42}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\rho[(C_n')^2]}{2}\right) = &-\frac{1}{\epsilon}\left\{\frac{\partial}{\partial y}\left(\rho\,\epsilon\left[\frac{(C_n')^2}{2}C_n\right] + \frac{\partial}{\partial z}\left(\rho\,\epsilon\left[\frac{(C_n')^2}{2}w\right]\right)\right\} - \\ &\rho[C_n'C_n']\frac{\partial[C_n]}{\partial y} - \rho[C_n'w']\frac{\partial[C_n]}{\partial z} - \rho f_n[C_T'C_n'] - \\ &\rho[C_T'C_n']\frac{[C_T]\cot\Delta\phi}{r} - \rho[C_T'C_n'C_T]\frac{\cot\Delta\phi}{r} - \\ &[C_n'\frac{\partial p'}{\partial y}] + \rho[C_n'D_y']\end{aligned} \tag{43}$$

Integrating (42) and (43) over V we obtain, finally,

$$\frac{\partial \mathbf{K}'}{\partial t} = \int_{S_a} [\frac{\rho(C_T')^2}{2} C_n]dS_a - \int_{S_b} [\frac{\rho(C_T')^2}{2} C_n]dS_b +$$

$$\int_{S_1} [\frac{\rho(C_T')^2}{2} w]dS_1 - \int_{S_2} [\frac{\rho(C_T')^2}{2} w]dS_2 -$$

$$\int_V (\rho[C_T'C_n'] \frac{\partial[C_T]}{\partial y} + \rho[C_T'w'] \frac{\partial[C_T]}{\partial z})dV + \int_V \rho f_n[C_T,C_n']dV +$$

$$\int_V \rho[C_T'C_T'] \frac{[C_n] \cot \Delta\phi}{r} dV + \int_V \rho[C_T'C_n'C_T] \frac{\cot \Delta\phi}{r} dV -$$

$$\int_V [C_T' \cdot \frac{\partial p'}{\partial x}]dV + \int_V \rho[C_T'D_x']dV \quad , \tag{44}$$

$$\frac{\partial \mathbf{k}'}{\partial t} = \int_{S_a} [\frac{\rho(C_n')^2}{2} C_n]dS_a - \int_{S_b} [\frac{\rho(C_n')^2}{2} C_n]dS_b +$$

$$\int_{S_1} [\frac{\rho(C_n')^2}{2} w]dS_1 - \int_{S_2} [\frac{\rho(C_n')^2}{2} w]dS_2 -$$

$$\int_V (\rho[C_n'C_n'] \frac{\partial[C_n]}{\partial y} + \rho[C_n'w'] \frac{\partial[C_n]}{\partial z})dV - \int_V \rho f_n[C_T'C_n']dV -$$

$$\int_V \rho[C_T'C_n] \frac{[C_T] \cot \Delta\phi}{r} dV - \int_V \rho[C_T'C_n'C_T] \frac{\cot \Delta\phi}{r} dV -$$

$$\int_V [C_n' \frac{\partial p'}{\partial y}]dV + \int_V \rho[C_n'D_y']dV \tag{45}$$

in which the various terms have the following interpretations:

$[\mathbf{K}'] \equiv \int_V \frac{\rho[(C_T')^2]}{2} dV$ is the total kinetic energy of the horizontal-eddy tangential flow contained within the volume.

$[\mathbf{k}'] \equiv \int_V \frac{\rho(C_n')^2}{2} dV$ is the total kinetic energy of the horizontal-eddy normal flow contained within the volume.

$$\int_{S_a} [\frac{\rho(C_T')^2}{2} C_n]dS_a - \int_{S_b} [\frac{\rho(C_T')^2}{2} C_n]dS_b + \int_{S_1} [\frac{\rho(C_T')^2}{2} w]dS_1 -$$

$\int_{S_2} [\frac{\rho(C_T')^2}{2} w]dS_2$ is the net transport into the volume of the kinetic energy of horizontal-eddy tangential flow.

$$\int_{S_a} [\frac{\rho(C_n')^2}{2} C_n]dS_a - \int_{S_b} [\frac{\rho(C_n')^2}{2} C_n]dS_b + \int_{S_1} [\frac{\rho(C_T')^2}{2} w]dS_1 -$$

$\int_{S_2} \frac{\rho(C_n')^2}{2} wdS_2$ is the net transport into the volume of the kinetic energy of horizontal-eddy normal flow.

$- \int_V (\rho[C_T'C_n'] \frac{\partial[C_T]}{\partial y} + \rho[C_T'w'] \frac{\partial[C_T]}{\partial z})dV$ is the total rate of generation of horizontal-eddy tangential kinetic energy within the volume by the horizontal-eddy stresses. The integrand of this term is given by the product of the rate of transport of tangential linear momentum across an elementary annular ring with the rate of shear of the azimuthal-mean tangential velocity across the ring. It is readily seen that a horizontal-eddy transport of momentum in the direction of the gradient of the azimuthal-mean flow will tend to increase the horizontal-eddy kinetic energy in the ring. This term is somewhat analogous to the pressure work term, $- \int_V [p] \frac{\partial[C_n]}{\partial y} dV$, in that the integrand represents a net rate of distortion of the ring by the mean motion acting against a stress. In the case of the pressure work term, a compression against the (normal) pressure stress is involved, whereas here a shear against a horizontal-eddy stress in the tangential direction is involved.

$$-\int_V \left(\rho[C_n'C_n'] \frac{\partial[C_n]}{\partial y} + \rho[C_n'w'] \frac{\partial[C_n]}{\partial z}\right) dV$$ is the total rate of generation of horizontal-eddy normal kinetic energy within the volume by the horizontal-eddy stresses. The integrand of this term is given by the product of the rate of transport of normal linear momentum across an elementary annular ring with the rate of compression on the ring due to the azimuthal-mean normal velocity. It is readily seen that a horizontal-eddy transport of momentum in the direction of the gradient of the azimuthal-mean flow will tend to increase the horizontal-eddy kinetic energy in the ring. This term is closely analogous to the pressure work term, $-\int_V [p] \frac{\partial[C_n]}{\partial y} dV$, in that the integrands of both represent a net rate of compression acting against a normal stress.

$$\int_V \rho f_n [C_T'C_n'] dV + \int_V \rho[C_T'C_T'] \frac{[C_n] \cot \Delta\phi}{r} dV + \int_V \rho[C_T'C_n'C_T'] \frac{\cot \Delta\phi}{r} dV$$

is the total rate of generation of horizontal-eddy tangential kinetic energy within the volume by the fictitious forces, through the agency of horizontal-eddy motions.

$$-\int_V \rho f_n [C_T'C_n'] dV - \int_V \rho[C_T'C_n'] \frac{[C_T] \cot \Delta\phi}{r} dV - \int_V \rho[C_T'C_T'C_T] \frac{\cot \Delta\phi}{r} dV$$

is the total rate of generation of horizontal-eddy normal kinetic energy within the volume by the fictitious forces, through the agency of horizontal-eddy motions.

$\int_V \rho[C_T'D_x'] dV$ is the total rate of generation of horizontal-eddy tangential kinetic energy within the volume by friction.

$\int_V \rho[C_n'D_y'] dV$ is the total rate of generation of horizontal-eddy normal kinetic energy within the volume by friction.

$-\int_V [C_T' \frac{\partial p'}{\partial x}] dV$ is the total rate of generation of horizontal-eddy tangential kinetic energy within the volume by the horizontal pressure forces.

$-\int_V [C_n' \frac{\partial p'}{\partial y}] dV$ is the total rate of generation of horizontal-eddy normal kinetic energy within the volume by the horizontal pressure forces.

5. ENERGY CYCLES OF ROTATING WIND SYSTEMS

With the use of the following notation,

$$\mathbf{T} = \int_S \frac{\rho[c_T]^2}{2} c_n dS$$

$$\mathbf{T}_w = \int_{S_1} \frac{\rho[c_T]^2}{2} w dS_1 - \int_{S_2} \frac{\rho[c_T]^2}{2} w dS_1$$

$$\mathbf{T}_n = \int_S \frac{\rho[c_n]^2}{2} c_n dS$$

$$\mathbf{T}_{nw} = \int_{S_1} \frac{\rho[c_n]^2}{2} w dS_1 - \int_{S_2} \frac{\rho[c_n]^2}{2} w dS_2$$

$$\mathbf{T}' = \int_S [\rho \frac{(c_T')^2}{2} c_n] dS$$

$$\mathbf{T}_w' = \int_{S_1} [\frac{\rho(c_T')^2}{2} w] dS_1 - \int_{S_2} [\frac{\rho(c_T')^2}{2} w] dS_2$$

$$\mathbf{T}_n' = \int_S [\frac{\rho(c_n')^2}{2} c_n] dS$$

$$\mathbf{T}_{nw}' = \int_{S_1} [\frac{\rho(c_n')^2}{2} w] dS_1 - \int_{S_2} [\frac{\rho(c_n')^2}{2} w] dS_2$$

$$\mathbf{E} = - \int_V \frac{[c_T]}{\epsilon} \left\{ \frac{\partial}{\partial y}(\rho \, \epsilon \, [c_T'c_n']) \right\} dV$$

$$\mathbf{E}_w = -\int_V \frac{[C_T]}{\epsilon}\left\{\frac{\partial}{\partial z}(\rho\,\epsilon\,[C_T'w'])\right\}\,dV$$

$$\mathbf{E}_n = -\int_V \frac{[C_n]}{\epsilon}\left\{\frac{\partial}{\partial y}(\rho\,\epsilon\,[C_n'C_n'])\right\}\,dV$$

$$\mathbf{E}_{nw} = -\int_V \frac{[C_n]}{\epsilon}\left\{\frac{\partial}{\partial z}(\rho\,\epsilon\,[C_n'w'])\right\}\,dV$$

$$\mathbf{E}' = -\int_V \rho[C_T'C_n']\,\frac{\partial[C_T]}{\partial y}\,dV$$

$$\mathbf{E}_w' = -\int_V \rho[C_T'w']\,\frac{\partial[C_T]}{\partial z}\,dV$$

$$\mathbf{E}_n' = -\int_V \rho[C_n'C_n']\,\frac{\partial[C_n]}{\partial y}\,dV$$

$$\mathbf{E}_{nw}' = -\int_V \rho[C_n'w']\,\frac{\partial[C_n]}{\partial z}\,dV$$

$$\mathbf{M} = \int_V \frac{\rho[C_T][C_T][C_n]\,\cot\Delta\phi}{r}\,dV$$

$$\mathbf{M}' = \int_V \frac{\rho[C_T][C_T'C_n']\,\cot\Delta\phi}{r}\,dV$$

$$\mathbf{M}'' = \int_V \frac{\rho[C_n][C_T'C_T']\,\cot\Delta\phi}{r}\,dV$$

$$\mathbf{M}''' = \int_V \rho[C_T'C_n'C_T]\,\frac{\cot\Delta\phi}{r}\,dV$$

$$\mathbf{C} = \int_V \rho f_n[C_T][C_n]dV$$

$$\mathbf{C}' = \int_V \rho f_n [C_T' C_n'] \, dV$$

$$\mathbf{W}_n = - \int_V [C_n] \frac{\partial [p]}{\partial y} \, dV$$

$$\mathbf{W}' = - \int_V [C_T' \frac{\partial p'}{\partial x}] \, dV$$

$$\mathbf{W}_n' = - \int_V [C_n' \frac{\partial p'}{\partial y}] \, dV$$

$$\widetilde{\mathbf{W}}_n = - \int_V [p] \frac{\partial [C_n]}{\partial y} \, dV$$

$$\widetilde{\mathbf{W}}' = - \int_V [p' \frac{\partial C_T'}{\partial x}] \, dV$$

$$\widetilde{\mathbf{W}}_n' = - \int_V [p' \frac{\partial C_n'}{\partial y}] \, dV$$

$$\mathbf{D} = \int_V \rho [C_T][D_x] \, dV$$

$$\mathbf{D}_n = \int_V \rho [C_n][D_y] \, dV$$

$$\mathbf{D}' = \int_V \rho [C_T' D_x'] \, dV$$

$$\mathbf{D}_n' = \int_V \rho [C_n' D_y'] \, dV$$

we may write equations (32), (33), (44), (45), and (22) in the forms shown on the following page:

$$\frac{\partial [K]}{\partial t} = (E + E_w) + (M + C) + M' + D_T + (T + T_w) \qquad (46)$$

$$\frac{\partial [k]}{\partial t} = (E_n + E_{nw}) - (M + C) - M'' + W_n + D_n + (T_n + T_{nw}) \qquad (47)$$

$$\frac{\partial K'}{\partial t} = (E' + E_w') + M'' + (M''' + C') + W' + D' + (T' + T_w) \qquad (48)$$

$$\frac{\partial k'}{\partial t} = (E_n' + E_{nw}') - M' - (M''' + C') + W_n' + D_n' + (T_n' + T_{nw}') \qquad (49)$$

$$\frac{\partial P}{\partial t} = \widetilde{W}_n + \widetilde{W}' + \widetilde{W}_n' + |\Psi_T| + |\Psi_n| + |\Psi'| + |\Psi_n'| \qquad (50)$$

where $|\Psi|$, $|\Psi_n|$, $|\Psi'|$, and $|\Psi_n|'$ represent rates of generation of heat by friction associated with azimuthal-mean tangential, azimuthal-mean normal, horizontal-eddy tangential, and horizontal-eddy normal motions, respectively.

In these equations, the E's are associated with the horizontal-eddy stresses, the M's and C's with the fictitious forces (the latter being the Coriolis terms), the W's with the horizontal pressure forces, the D's and $|\Psi|$'s with friction, and the T's are rates of energy transport into the volume.

It will be noted that the M's and C's each occur in two different equations with opposite signs. Since they are not separated by boundary terms they therefore measure conversions among the various forms of energy. In particular, they represent conversions between normal and tangential forms of kinetic energy. The E's, W's, D's, and $|\Psi|$'s, on the other hand, are separated by boundary terms, as follows:

$$E = E' + E''$$

$$E_w = E_w' + E_w''$$

$$E_n = E_n' + E_n''$$

$$E_{nw} = E_{nw}' + E_{nw}''$$

$$W_n = \widetilde{W}_n + \approx{W}_n$$

$$W' = \widetilde{W}' + \approx{W}'$$

$$W_n' = \widetilde{W}_n' + \approx{W}_n'$$

$$D_T = |\Psi|_T + S_T$$

$$D_n = |\Psi|_n + S_n$$

$$D' = |\Psi|' + S'$$

$$D_n' = |\Psi|_n' + S_n'$$

where

$$E'' = \int_S \rho[C_T][C_T'C_n'] \, dS$$

$$E_w'' = \int_{S_1} \rho[C_T][C_T'w'] \, dS_1 - \int_{S_2} \rho[C_T][C_T'w'] \, dS_2$$

$$\mathbf{E}_n'' = \int_S \rho[c_n][c_n'c_n'] \, dS$$

$$\mathbf{E}_{nw}'' = \int_{S_1} \rho[c_n][c_n'w'] \, dS_1 - \int_{S_2} \rho[c_n][c_n w'] \, dS_2$$

$$\widetilde{\widetilde{\mathbf{W}}}_n = \int_S [c_n][p] \, dS$$

$$\widetilde{\widetilde{\mathbf{W}}}' = 0$$

$$\widetilde{\widetilde{\mathbf{W}}}_n' = \int_S c_n'p' dS$$

Therefore, only when the corresponding boundary terms vanish identically may each one be regarded as a measure of the rate of conversion between two forms of energy. The arrows in equations (46) through (50) represent these conversions. (It will be noted that the conversions associated with friction may proceed in only one direction, namely from kinetic to potential plus internal energy.)

According to equations (46) through (50), the only form of energy directly affected by heating, $\mathbf{H}$, is internal plus potential energy. This may, in turn, be converted to horizontal-eddy forms of kinetic energy, or to azimuthal-mean normal kinetic energy, through work done by the horizontal pressure forces. Such conversions are represented by the $\mathbf{W}$'s. There is, however, no direct link between internal plus potential energy and azimuthal-mean tangential kinetic energy. The latter must, therefore, receive its energy either from azimuthal-mean normal kinetic energy, through conversions represented by ($\mathbf{M}$ + $\mathbf{C}$), or from horizontal-eddy kinetic energy, through conversions represented by ($\mathbf{E} + \mathbf{E}_w$) and $\mathbf{M}$. (It may be noted that the $\mathbf{M}$'s are important only in systems such as the hurricane, in which $[c_T]$ is intense near the axis. In the case of larger-scale systems, such as the general circulation, these terms are negligible.) If we combine equations (48) and (49) and think of $\mathbf{K}' + \mathbf{k}'$ as a single form of energy,

$$\frac{\partial [K]}{\partial t} = \boxed{(E + E_w) + M'} + \boxed{M + C} + \boxed{D_T} \qquad (51)$$

$$\frac{\partial [k]}{\partial t} = -\boxed{M + C} + \boxed{(E_n + E_{nw}) - M''} + \boxed{W_n} + \boxed{D_n} \qquad (52)$$

$$\partial (K' + k) = \boxed{(E' + E_w') - M'} \quad \boxed{(E_n' + E_{nw}') + M''} + \boxed{W' + W_n'} + \boxed{D' + D_n'} \qquad (53)$$

$$\frac{\partial P}{\partial t} = + \boxed{\widetilde{W}_n} + \boxed{\widetilde{W}' + \widetilde{W}_n'} + \boxed{|\Psi_T|} + \boxed{|\Psi_n|} + \boxed{|\Psi|' + |\Psi_n|'} + H \qquad (54)$$

we may distinguish several possible cycles which can lead to the maintenance of the kinetic energy of azimuthal-mean rotation:

1) P is converted to $[\,\mathbf{k}\,]$ through work done by the horizontal pressure forces (W_n). $[\,\mathbf{k}\,]$ is, in turn, converted to $[\,\mathsf{K}\,]$ through work done by the fictitious forces ($\mathsf{M} + \mathsf{C}$).

2) P is converted to $\mathsf{K}' + \mathbf{k}'$ through work done by the horizontal pressure forces ($\mathsf{W}' + \mathsf{W}_n'$). $\mathsf{K}' + \mathbf{k}'$ is, in turn, converted to $[\,\mathsf{K}\,]$ through work done by the eddy stresses ($\mathsf{E}' + \mathsf{E}_w'$) and by the fictitious forces (M').

3) P is converted to $[\,\mathbf{k}\,]$ through work done by the horizontal pressure forces (W_n). $[\,\mathbf{k}\,]$ is, in turn, converted to $\mathsf{K}' + \mathbf{k}'$ through work done by the eddy stresses ($\mathsf{E}_n' + \mathsf{E}_{nw}'$) and by the fictitious forces (M''). Finally, $\mathsf{K}' + \mathbf{k}'$ is converted to $[\,\mathsf{K}\,]$ through work done by the eddy stresses ($\mathsf{E}' + \mathsf{E}_w'$) and by the fictitious forces (M').

4) P is converted to $[\mathbf{k}]$ through work done by the horizontal pressure forces ($\mathsf{W}' + \mathsf{W}_n'$). $\mathsf{K}' + \mathbf{k}'$ is, in turn, converted to $[\,\mathbf{k}\,]$ through work done by the eddy stresses ($\mathsf{E}_n' + \mathsf{E}_{nw}'$) and by the fictitious forces (M''). Finally, $[\mathbf{k}]$ is converted to $[\,\mathsf{K}\,]$ through work done by the fictitious forces ($\mathsf{M} + \mathsf{C}$).

5) Various combinations of the above cycles may also lead to the maintenance of $[\,\mathsf{K}\,]$.

The different energy cycles presented above can be traced with the aid of figure 1. Among the above possibilities the first two are the more familiar ones. The first is characteristic of the so-called "Hadley regime" of convection, which develops in rotating model experiments (e.g., Fultz [5]) when the radial gradient of the heating is intense and the rate of rotation of the vessel is small. The second is characteristic of the "Rossby regime" of convection, which develops under conditions of weak heating and high rotation. By analogy, it would be expected that rotating systems in the earth's atmosphere which occur under different conditions of heating and rotation are maintained by characteristically different energy cycles. Let us examine the cycles associated with a few of these rotating wind systems:

We consider first the largest-scale rotating system in the atmosphere, namely, the average zonal wind current which encircles each hemisphere. In this circulation the most intense westerlies and easterlies are located sufficiently far from the pole that the M's in equations (46) through (50) are negligible. Furthermore, as shown by Kuo [15], the observed north-south temperature contrast is not great enough, relative to the rate of rotation of the earth about its polar axis, to drive a general circulatinn of the Hadley type. Thus, conversions of the type C must be small. It turns out that potential plus internal energy is converted first into the kinetic energy of horizontal eddies which, in this case, are the "waves in the westerlies". Horizontal-eddy kinetic energy is then converted to mean zonal kinetic energy

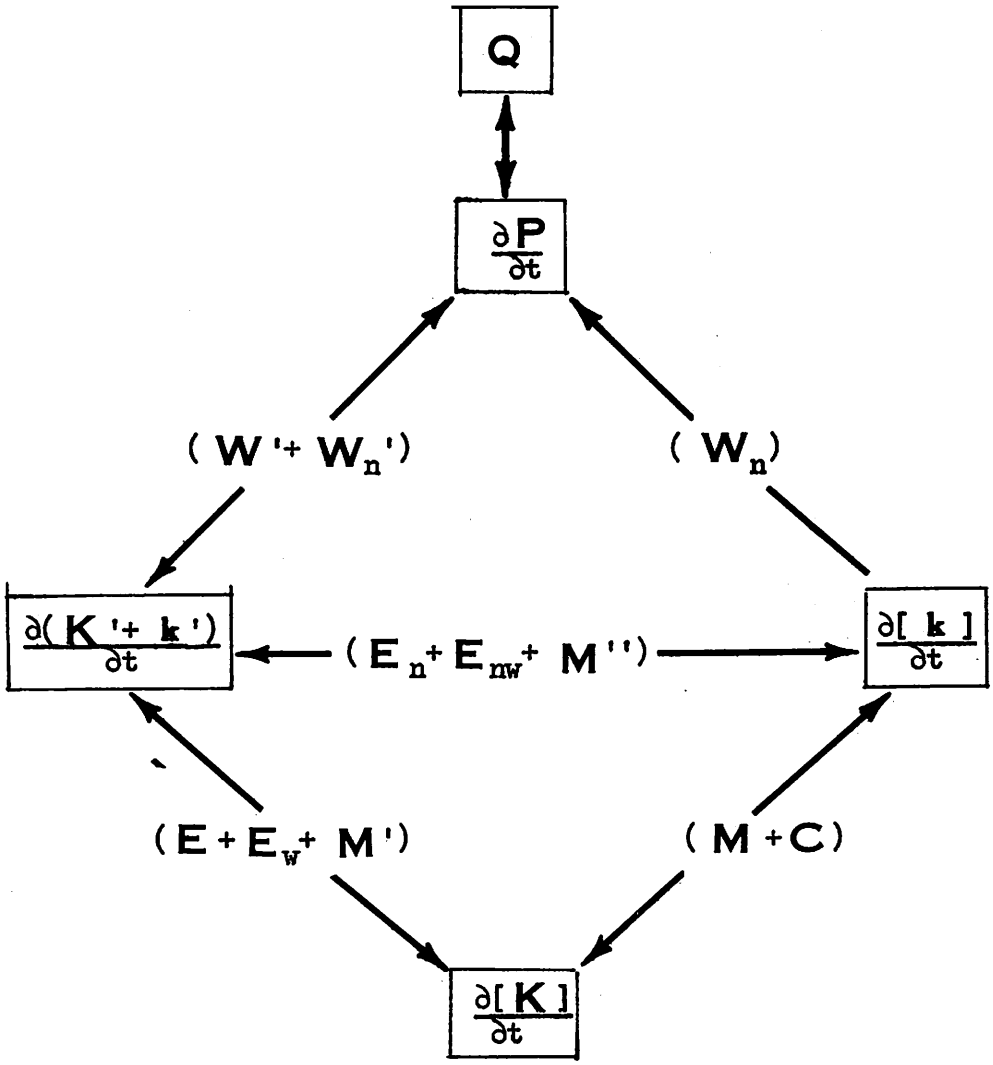

Figure 1. - Energy cycles of rotating wind systems.

through conversions of the type $(E' + E_w')$. (It may be noted that the direction of the latter conversion is exactly opposite to that which would be expected in the case of small-scale horizontal-eddy viscosity.) In addition, there is some evidence (Starr [23]) to indicate that mean zonal kinetic energy is converted into mean meridional kinetic energy through the action of the Coriolis force (C). Although this conversion is small, it appears to be sufficient to maintain the extremely dry horse latitudes through sinking motions in the vicinity of 30°N. and S. latitude, and the belts of cloud and rain near 60°N. and S. latitude through rising motions in these belts. The energy cycle of the global wind circulation is summarized in figure 2.

Next, we consider the maintenance of extratropical disturbances. As

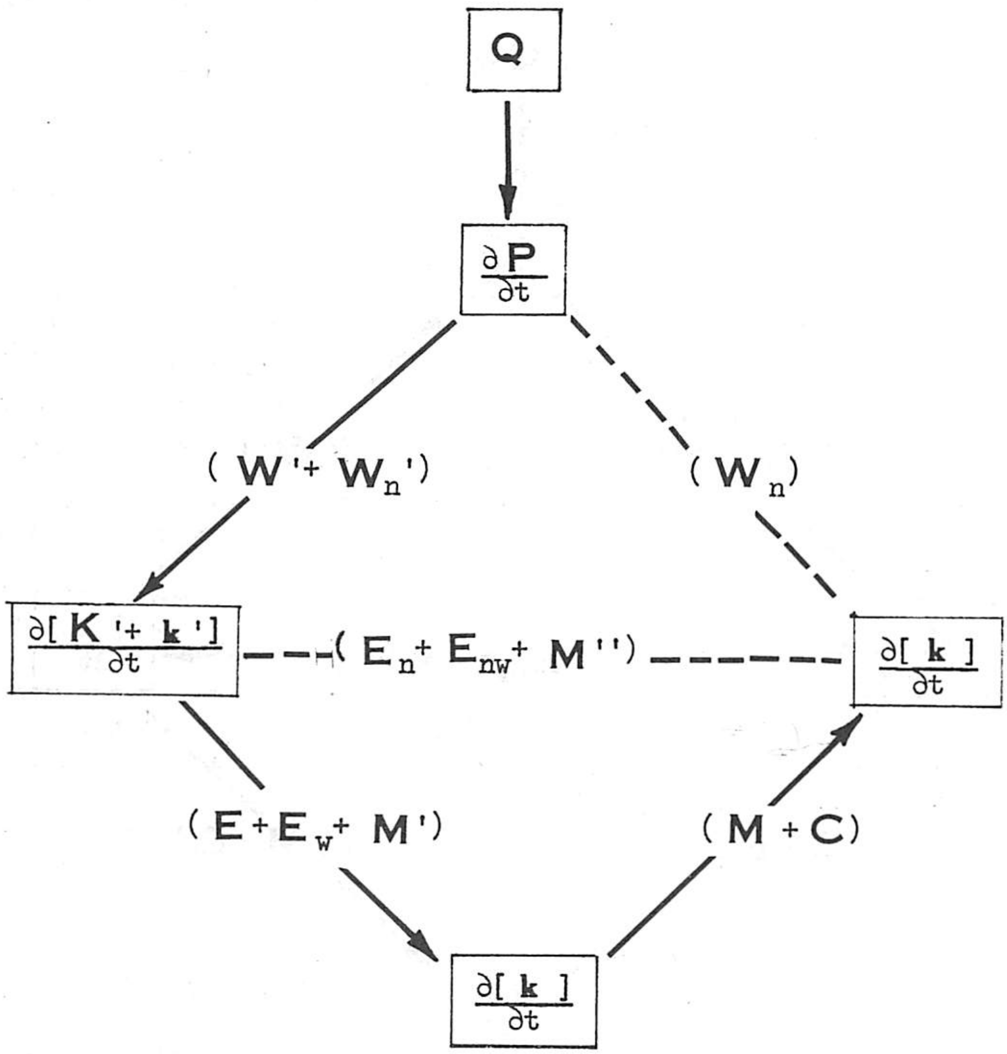

Figure 2. - Energy cycle of the general circulation of the earth's atmosphere.

noted above, these are the "horizontal-eddies" in the general circulation. Viewed as a group, these systems receive their kinetic energy primarily through conversions from potential plus internal energy. If, however, we focus attention on one such system, regarding its kinetic energy as the azimuthal-mean kinetic energy, and the kinetic energy of the surrounding systems as horizontal-eddy kinetic energy, we find that the system under consideration may interact with the neighboring systems through horizontal-eddy conversions of the form $(E + E_w')$ and M', and through various boundary terms. Referring to equations (46) through (50), we see that this implies that potential energy is being converted to both horizontal-eddy kinetic energy and to azimuthal-mean normal kinetic energy through work done by the horizontal pressure forces. It is not possible to say, a priori, in what direction the eddy conversions $(E' + E_w')$ and M' proceed in this case.

In particular, do any of the extratropical systems feed on the energy of others? If so, what are the characteristics of such systems, and what are the characteristics of the systems which give up their energy? Questions such as these can be answered only through an appeal to data.

We turn now to a consideration of the tropical cyclone. Since the component of the earth's rotation about a vertical axis located in the Tropics is small, the intensity of the heating required to produce a circulation of the Hadley type in the Tropics is not as great as that of circulation elsewhere on the globe. As it turns out, the hurricane has many characteristics of the Hadley regime. In particular, large conversions from [k] to [K] take place through the action of the processes $\mathsf{M} + \mathsf{C}$. Once again, however, the question arises whether there are conversions of the type $\mathsf{E}' + \mathsf{E}_w'$ and M', and in what direction they proceed. In this connection, it may be noted that hurricanes do not form as often as might be expected on the basis of the presence of conditional instability in the Tropics. Do hurricanes, in fact, require an additional source of rotational energy? Does this additional energy come from the neighboring systems through conversions of the type $\mathsf{E}' + \mathsf{E}_w'$ and M' ? Questions of this nature will be looked into in section 6.

Finally, in the case of the tornado, it is readily shown that conversions of the type C are entirely negligible. It would appear on the basis of scattered evidence that the tornado derives its rotational energy from intense shears in the thunderstorm, largely through the term M, although no quantitative measurements are available to confirm this speculation. The mechanism of the tornado is complicated, however, by the fact that the axis of rotation is not vertical. There must, therefore, be transports of the type T out of the region of kinetic energy generation.

6. ENERGY TRANSFORMATIONS IN THE MATURE HURRICANE

In order to answer questions such as those posed in section 5, it will be necessary to compute, from observational data, the direction and magnitude of each link in the energy cycle of each type of rotating system in which we are interested. In the present section a beginning is made in this direction in the case of the mature hurricane. Here, certain aspects of the energy cycle of the hurricane are examined quantitatively. Owing to the complexity of the problem, however, and the lack of adequate observational information about thermodynamical processes in the hurricane, this investigation deals only with the mechanical aspects of the problem. More specifically, only those processes which directly affect the state of rotation of the atmosphere are examined, and no direct information is given regarding the manner in which these processes are related to the thermodynamics. It is felt however, that a great deal of insight into the mechanism of the hurricane, or any other rotating system in the atmosphere, can be gained by examining even small portions of the energy cycle.

The data and computational procedures employed in this study were discussed in [18] and [19]. In accordance with the remarks in [19] concerning the measurability of the term, $[C_T'w']$, the integral E_w was not evaluated.

In the computations the quantity $[C_n]$ was replaced everywhere by $[C_n]''$. Estimates of the work done by friction were made on the basis of the approximate formula,

$$\mathbf{D}_T = -2\pi\, r^2 \rho_0 K \int_{\Delta\phi} [C_T]_0 [|C|\, C_T]_0 \sin \Delta\phi \, d(\Delta\phi) \quad , \qquad (55)$$

where the zero subscripts refer to quantities measured at 1000 ft., and the magnitude of the skin friction coefficient, K, and the density at 1000 ft., ρ_0, were taken as $K = .0015$ and $\rho_0 = 1.1903 \times 10^{-3}$ gm. cm.$^{-3}$. The trapezoidal rule was used to evaluate all integrals.

Owing to the limitations of the data, the numerical evaluation of the various integrals in (46) extends only over the region $2° < \Delta\phi < 6°$. Since the portion of the atmosphere contained within this volume cannot be regarded as a closed system, the boundary integrals, as well as the generation terms, must be considered.

Estimates of the various energy integrals which appear in equation (46) are given in the first five rows of table 1. The first two columns of this table provide a breakdown of the integrals for the inner $(2° < \Delta\phi < 4°)$ and outer $(4° < \Delta\phi < 6°)$ portions of the volume, respectively, while the last column gives the total for the volume. Focusing attention on the last column, we find that the generation of azimuthal-mean tangential kinetic energy by

Table 1. - Estimates of energy integrals in equation (46), based on composite wind charts of the mature hurricane. 1 unit = 10^{18} gm. cm.^{2}sec.$^{-3}$

$\Delta\phi$	2° - 4°	4° - 6°	2° - 6°
T	-11.1	-7.6	-18.8
C	+27.3	+25.1	+52.4
M	+26.8	+6.2	+33.0
M'	+1.6	+1.6	+3.2
E	+5.8	+2.3	+8.1
T + C + M	+42.97	+23.7	+66.6
M' + E	+7.40	+3.9	+11.3
Total	+50.37	+27.6	+77.9
Frictional drain	+45.11	+28.3	+73.4

azimuthal-mean motions, ($\mathsf{M} + \mathsf{C}$), is about eight times as large as the generatinn by horizontal-eddy processes, ($\mathsf{M}' + \mathsf{E}$). Since the azimuthal-mean motions serve also to transport energy out of the volume, however, their net contribution is about six times as large as that of the horizontal eddies. This may be seen by comparing rows 6 and 7 in the last column.

Turning now to the first two columns and comparing rows 6 and 7, we find that azimuthal-mean processes bear roughly the same ratio to horizontal-eddy processes in both portions of the volume. It may be noted, also, that both the azimuthal-mean and the horizontal-eddy processes provide a greater contribution in the inner region than they do in the outer region. For the horizontal-eddy processes this is just the reverse of what was found in the angular momentum balance (table 1 of [19]). This result is not surprising, however, when we realize that a convergence of linear momentum at high levels in the outer region, where the rotation is anticyclonic (shaded area in fig. 3 of [18]), leads to a negative value of E in this region.

Comparison of the values in the last two rows of table 1 indicates that the total contribution of the measured terms closely satisfies the frictional requirements. This is apparently true in each portion of the volume as well as in the entire volume. Thus, it is likely that the neglected integral, E_w, does not contribute a great deal to the energy balance of each region.

Tables 2 and 3 give a further breakdown of the results. In these tables the energy budgets of the lower and upper portions of the atmosphere are presented. The vertical velocities computed in [19] were used in the evaluation of T_w, and the assumption was made that friction acts to abstract kinetic energy from the lower layer and has no effect on the upper layer.

Comparison of the last two rows in each table reveals that the various generation terms and boundary integrals which have been measured supply too much energy to the upper portion of the atmosphere, and too little to the lower portion. This might be due to errors in the data, or it could be due to the neglect of the integral, E_w. If there were a vertical convergence of linear momentum in the lower layer, and a vertical divergence of linear momentum in the upper layer, this integral could bring about the required redistribution of energy in the vertical without affecting the energy budget of the volume as a whole.

It will be noted that the discrepancies between the last two rows are more serious in table 3 than they are in table 2, since the individual terms in the first part of the table are smaller in the case of the former. It might not be meaningful, therefore, to make detailed comparisons among the individual terms. Two features of these tables appear to be significant, however. In the first place, it may be noted that, in the upper portion of the atmosphere, the integral, E, is very much larger in the inner region than it is in the outer region. This may be explained by the fact that the convergence of linear momentum at high levels is weighted by a comparatively large positive value of $[C_T]$ in the inner region, and a negative, or small positive, value of $[C_T]$ in the outer region. Thus, a process that tends to increase the momentum in a region may tend to decrease the kinetic energy in the same region.

Table 2. - The energy balance of the lower portion of the atmosphere (surface to 600 mb.), based on the composite wind charts of the mature hurricane. 1 unit = 10^{18} gm. cm.^{2} sec.$^{-3}$

$\Delta \phi$	2° - 4°	4° - 6°	2° - 6°
T_w	0	0	0
T	-12.6	-5.0	-17.6
C	+25.7	+16.9	+42.5
M	+24.0	+6.3	+30.2
M'	+0.6	+1.1	+1.8
E	+1.6	+2.2	+3.9
$T_w + T + C + M$	+37.0	+18.1	+55.1
$M' + E$	+2.2	+3.4	+5.6
Total	+39.2	+21.5	+60.7
Frictional drain	+45.1	+28.3	+73.4

Table 3. - The energy balance of the upper portion of the atmosphere (600 mb. to 125 mb.) based on composite wind charts of the mature hurricane. 1 unit = 10^{18} gm. cm.^{2} sec.$^{-3}$

$\Delta \phi$	2° - 4°	4° - 6°	2° - 6°
T_w	0	0	0
T	+1.5	-2.6	-1.1
C	+1.6	+8.2	+9.8
M	+2.8	-0.1	+2.8
M'	+1.0	+0.4	+1.4
E	+4.2	+0.1	+4.3
$T_w + T + C + M$	+6.0	+5.5	+11.5
$M' + E$	+5.2	+0.5	+5.7
Total	+11.1	+6.1	+17.2
Frictional drain	0	0	0

It may be noted, in this connection, that even if the present measurements have underestimated the angular momentum convergence in the upper portion of the outer region, the qualitative result that E is small in this region is probably correct, since $[C_T]$ is negative in one part of the region and positive in the other.

The second feature of interest is that the processes associated with the azimuthal-mean motion take on much larger values in the lower layer than they do in the upper layer (compare row 7, column 3 of tables 2 and 3), whereas the magnitude of the horizontal-eddy processes is roughly the same in the two layers (compare row 8 column 3 of tables 2 and 3). The significance of this result is that one cannot expect the ratio of two different physical processes, measured at a single level in the atmosphere, to be representative of the ratio of the integrals of these processes through the depth of the atmosphere. In the present case, the ratio of the contribution by horizontal-eddy processes to that by azimuthal-mear motions would be greatly overestimated in the upper portion of the atmosphere and underestimated in the lower portion of the atmosphere.

It is of interest to compare the variation of M and C with distance from the center of the hurricane. Returning to table 1 we find that M is much larger in the inner region than it is in the outer region, whereas C increases only slightly from the outer to the inner region. It should be noted in this connection, that the ratio of M to C is of the order

$$\frac{[\,C_T\,]}{(r \sin \Delta\phi)\omega_{\phi_0}} .$$

Therefore, since $[C_T]$ increases toward the center of the hurricane, while $(r \sin \Delta\phi)\omega_{\phi_0}$ decreases, M must become larger than C as we approach the center of the hurricane. Linear extrapolation of the values in table 1 would give for the volume $0° < \Delta\phi < 2°$, $\mathsf{M} \sim +50 \times 10^{18}$ gm. cm.^{2}sec.$^{-3}$ (which is probably an underestimate, since M depends on the square of the azimuthal-mean tangential velocity), and $\mathsf{C} \sim +29 \times 10^{18}$ gm. cm.^{2}sec.$^{-3}$. This may be compared with the transport, T, and the work done by the horizontal-eddy stresses, E, across $\Delta\phi = 2°$, which are measured to be approximately $+23 \times 10^{18}$ gm. cm.^{2}sec.$^{-3}$ and 1×10^{18} gm. cm.^{2}sec.$^{-3}$, respectively. Although there is no way of estimating the magnitude of the term, E, it is probably safe to say that for the region $0° < \Delta\phi < 2°$ this integral is negligible in comparison with M. This follows from the observation that the flow patterns around a hurricane become more symmetrical near the center of the storm, and also that, with decreasing distance from the center of the hurricane, the major contribution to the convergence of the horizontal-eddy flux of momentum shifts to higher levels, where the weighting by $[C_T]$ is smaller.

Estimates of the generation terms, E and E' and the boundary integral, E'', are given in table 4. The values in the last column show that

Table 4. - Estimates of the horizontal-eddy terms, based on composite wind charts of the mature hurricane. 1 unit = 10^{18} gm. cm.^{2} sec.$^{-3}$

$\Delta\phi$	2° - 4°	4° - 6°	2° - 6°
E	+5.8	+2.3	+8.1
E'	-2.8	-3.8	-6.5
E''	+3.0	-1.4	+1.6

the horizontal-eddies are tending to lose, and the azimuthal-mean flow to gain, energy through the action of the large-scale eddy stresses within the volume. Furthermore, the magnitudes of the two processes are not very different. In accordance with the conclusions reached in another article (Pfeffer [20]) it would not be correct to speak of a rate of conversion between horizontal-eddy and azimuthal-mean components of kinetic energy within this volume, since the <u>integrand</u> of E'' does not vanish. In this connection, it may be noted that the value of E'' on the outer boundary ($\Delta\phi$ = 6°) is 3.7 x 10^{18} gm. cm.^{2} sec.$^{-3}$. Thus, it is not possible to determine the direction in which the energy cycle proceeds in this case. It is clear, however, that the horizontal-eddy stresses give a negative generation of horizontal-eddy energy and a positive generation of mean tangential kinetic energy.

Focusing attention on the first two columns in table 4 we find that the maximum negative generation of horizontal-eddy energy takes place in the outer region, whereas the maximum positive generation of azimuthal-mean tangential kinetic energy takes place in the inner region. If the E 's do, in fact, represent conversions between K and [K], then it is apparent that the azimuthal-mean rotation within the inner region is gaining energy at the expense of the horizontal eddies in the outer region.

The analysis of the horizontal-eddy processes by layers is shown in tables 5 and 6.

Table 5. - Estimates of the horizontal-eddy terms within the upper layer, based on composite wind charts of the mature hurricane. 1 unit = 10^{18} gm. cm.^{2} sec.$^{-3}$

$\Delta\phi$	2° - 4°	4° - 6°	2° - 6°
E	+ 4.2	+0.1	+4.3
E'	-2.1	-2.2	-4.2
E''	+2.1	-2.1	+0.0

Table 6. - Estimates of the horizontal-eddy terms within the lower layer based on composite wind charts of the mature hurricane. 1 unit = 10^{18} gm. cm.^{2} sec.$^{-3}$

$\Delta\phi$	2° - 4°	4° - 6°	2° - 6°
E	+1.6	+2.2	+3.9
E'	-0.7	-1.6	-2.3
E''	+1.0	+0.6	+1.6

The large difference in E between the inner and outer regions in the upper layer was noted earlier. Focusing attention, now, on the second row of each table, we find that the absolute value of E' is larger in the upper layer than in the lower layer. This is in accordance with the fact that the steepest radial gradients of $[C_T]$ are found at high levels (fig. 3 of [18]). Another interesting feature that may be noted (row 3 of tables 5 and 6) is that the boundary integral E'' vanishes in the upper layer but not in the lower layer. Evidently, interactions with neighboring systems extend to greater distances from the hurricane center in the lower layers of the atmosphere.

7. CONCLUSIONS

It has long been recognized that azimuthal-mean motions play an important role in the mechanism of the hurricane. From a hydrodynamical standpoint there could be little doubt that energy conversions of the type $(M + C)$ contribute significantly to the maintenance of rotational motion in the hurricane. Very little has been known, however, about the magnitude and direction of energy conversions of the type $(E + E_W)$ and M', which represent interactions between the hurricane and its surroundings. Since the hurricane apparently has its own driving force (viz., heating at the center) it might have been expected, a priori, that such interactions would be in the sense of a viscous effect, in which the energy of organized rotation tends to become dissipated into the energy of horizontal eddies. Perhaps the most significant feature of the present results is that they show the reverse to be the case; namely, that the hurricane actually feeds on the energy of the horizontal eddies [2].

Since the horizontal-eddy processes contribute only about one-seventh of the energy necessary to maintain the rotation in the volume considered, it might be asked whether, to a first approximation at least, we may neglect them in attempting to account for the genesis and maintenance of the hurricane. To be able to answer this question in the affirmative, it would be necessary to explain, without resorting to a horizontal-eddy mechanism, why it is that

2 Other investigators (see, for example, Arakawa [1] and Rodriguez Ramirez [22]) have speculated that this might be the case, but, to the writer's knowledge no actual measurements were made before the present study.

hurricanes appear as infrequently as they do, in the face of the rather frequent occurrence in the Tropics of widespread thunderstorm activity, accompanied by the release of large amounts of latent heat. In this connection, it may be noted that hurricanes almost always form within already present easterly waves or shear zones, which are apparently maintained by horizontal-eddy processes.

Viewed in the light of the fluid model experiments, the hurricane may be said to possess characteristics of both the Hadley and the Rossby regimes of convection.

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